The following question paper is based only on the syllabus for Semester 2 examination. To test your knowledge on the remaining syllabus, kindly visit our website for an exhaustive MCQ based question bank.

SEMESTER 2 EXAMINATION 2022
MATHEMATICS

Maximum Marks : 40

Time allowed : One and a half hours.

Answers to this Paper must be written on the paper provided separately.

You will not be allowed to write during the first 10 minutes.

This time is to be spent in reading the question paper.

The time given at the head of this Paper is the time allowed for writing the answers.

Attempt all questions from Section A and any three questions from Section B.

The marks intended for questions are given in brackets [ ].

Mathematical tables are provided.

SECTION A
*(Attempt **all** questions)*

Question **1** :

Choose the correct answers to the questions from the given options. (Do not copy the question. Write the correct answer only.) **[10]**

(i) The probability of getting a number divisible by 3 in throwing a dice is :

   (a) $\dfrac{1}{6}$ (b) $\dfrac{1}{3}$ (c) $\dfrac{1}{2}$ (d) $\dfrac{2}{3}$

Ans. (b) $\dfrac{1}{3}$

   Explanation :

      Number of favourable cases = 2(3 or 6)

      and total number of cases = 6(1, 2, 3, 4, 5 or 6)

   ∴ **Required probability** $= \dfrac{2}{6} = \dfrac{1}{3}$

(ii) The volume of a conical tent is 462 m^3 and the area of the base is 154 m^2. The height of the cone is :

   (a) 15 m   (b) 12 m   (c) 9 m   (d) 24 m

Ans. (c) 9 m

   Explanation :

   Given : $\dfrac{1}{3}\pi^2 rh = 462$ m^3 and $\pi r^2 = 154$ m^2

   $\Rightarrow \dfrac{1}{3} \times 154 \times h = 462$ and $h = \dfrac{462 \times 3}{154}$ m = 9 m

   ∴ **The height of the cone = 9 m**

(iii) The median class for the given distribution is :

Class Interval	0-10	10-20	20-30	30-40
Frequency	2	4	3	5

    (a) 0-10   (b) 10-20   (c) 20-30   (d) 30-40

Ans. (c) Median class is 20-30

Explanation :

C.I.	Frequency	Cumulative frequency
0-10	2	2
10-20	4	6
20-30	3	9
30-40	5	14

$$\text{Median} = \frac{1}{2}\left[\frac{14^{th} \text{ term}}{2} + \left(\frac{14}{2}+1\right)^{th} \text{ term}\right]$$

$$= \frac{1}{2}\left[7^{th} \text{ term} + 8^{th} \text{ term}\right]$$

7th term and 8th term both belong to class-interval 20-30.

∴ **Required median class is 20-30.**

(iv) If two lines are perpendicular to one another then the relation between their slopes m_1 and m_2 is :

    (a) $m_1 = m_2$ (b) $m_1 = \dfrac{1}{m_2}$ (c) $m_1 = -m_2$ (d) $m_1 \times m_2 = -1$

Ans. (d) $m_1 \times m_2 = -1$

Explanation :

If m_1 is the slope of a line and m_2 is the slope of its perpendicular

$$m_1 = \frac{-1}{m_2} \quad \Rightarrow \quad m_1 \times m_2 = -1$$

∴ **Required relation is $m_1 \times m_2 = -1$**

(v) A lighthouse is 80 m high. The angle of elevation of its top from a point 80 m away from its foot along the same horizontal line is :

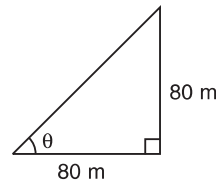   (a) 60°   (b) 45°   (c) 30°   (d) 90°

Ans. (b) 45°

Explanation :

If angle of elevation is θ,

then $\tan \theta = \dfrac{80 \text{ m}}{80 \text{ m}} = 1 = \tan 45°$

⇒ **Angle of elevation, θ = 45°.**

(vi) The modal class of a given distribution always corresponds to the :

    (a) interval with highest frequency   (b) interval with lowest frequency

    (c) the first interval   (d) the last interval

Ans. (a) interval with highest frequency

Explanation :

The mode of the given data belongs to the interval with highest frequency so modal class (class containing mode) always corresponds to the interval with highest frequency.

(vii) The co-ordinates of the point P(–3, 5) on reflecting on the x-axis are :

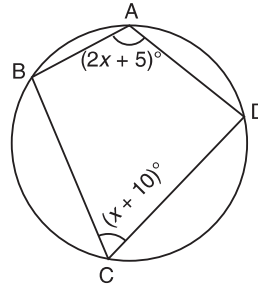   (a) (3, 5)   (b) (–3, –5)   (c) (3, –5)   (d) (–3, 5)

Ans. (b) (–3, –5)

    Explanation :

    The co-ordinates of the point (x, y) on reflection on the x-axis are $(x, -y)$.

    ∴ The co-ordinates of the point P(–3, 5) on reflection on the x-axis are **(–3, –5)**

(viii) ABCD is a cyclic quadrilateral. If ∠BAD = $(2x + 5)°$ and ∠BCD = $(x + 10)°$ then x is equal to :

    (a) 65°   (b) 45°

    (c) 55°   (d) 5°

Ans. (c) 55°

    Explanation :

    Since the opposite angles of a cyclic quadrilateral are supplementary,

       ∠A + ∠C = 180°   ⇒ $(2x + 5)° + (x + 10)° = 180°$

    i.e. $3x + 15 = 180$ ⇒   **$x = 55$**

(ix) A(1, 4), B(4, 1) and C(x, 4) are the vertices of ΔABC. If the centroid of the triangle is G(4, 3) then x is equal to :

    (a) 2   (b) 1   (c) 7   (d) 4

Ans. (c) 7

    Explanation :

    G(4, 3) is the centroid of triangle ABC, with A = (1, 4), B = (4, 1) and C = (x, 4)

    ⇒ $\dfrac{1 + 4 + x}{3} = 4$ *i.e.* $5 + x = 12$ and **$x = 7$**

(x) The radius of a roller 100 cm long is 14 cm. The curved surface area of the roller is : (Take $\pi = \dfrac{22}{7}$)

    (a) 13200 cm²   (b) 15400 cm²   (c) 4400 cm²   (d) 8800 cm²

Ans. (d) 8800 cm²

    Explanation :

    Roller is in the shape of a cylinder with length 100 cm and radius 14 cm.

    Curved surface area of the roller = Curved surface area of the cylinder

$$= 2\pi \times r \times h$$

$$= 2 \times \frac{22}{7} \times 14 \times 100 \text{ cm}^2$$

$$= \textbf{8800 cm}^2$$

SECTION B

*(Attempt **any three** questions from this Section)*

Question 2 :

(i) Prove that : **[2]**

$$\frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta} = 2\sec^2\theta$$

Ans. L.H.S $= \dfrac{1}{1+\sin\theta} + \dfrac{1}{1-\sin\theta}$

$$= \frac{1-\sin\theta + 1+\sin\theta}{(1+\sin\theta)\,(1-\sin\theta)}$$

$$= \frac{2}{1-\sin^2\theta} = \frac{2}{\cos^2\theta} = \mathbf{2\ sec^2\ \theta = R.H.S.} \qquad \textbf{Hence proved.}$$

(ii) Find '*a*', if A($2a + 2$, 3), B(7, 4) and C($2a + 5$, 2) are collinear. **[2]**

Ans. Points A($2a + 2$, 3), B(7, 4) and C($2a + 5$, 2) are collinear.

$\Rightarrow$ Slope of line AB = Slope of line BC

$\Rightarrow \qquad \dfrac{4-3}{7-2a-2} = \dfrac{2-4}{2a+5-7}$

$\Rightarrow \qquad \dfrac{1}{5-2a} = \dfrac{-2}{2a-2}$ *i.e.* $-10 + 4a = 2a - 2$

$\Rightarrow \qquad 2a = 8$ and $\boldsymbol{a = 4}$

(iii) Calculate the mean of the following frequency distribution. **[3]**

Class Interval	5-15	15-25	25-35	35-45	45-55
Frequency	2	6	4	8	4

Ans.

C.I.	f	x	$f \times x$
5-15	2	10	20
15-25	6	20	120
25-35	4	30	120
35-45	8	40	320
45-55	4	50	200
$\Sigma f = 24$		$\Sigma fx = 780$	

Mean $= \dfrac{\Sigma f \times x}{\Sigma f}$

$= \dfrac{780}{24} = \mathbf{32{\cdot}5}$

(iv) In the given figure O is the centre of the circle. PQ and PR are tangents and $\angle$QPR = 70°. Calculate : **[3]**

(a) $\angle$QOR

(b) $\angle$QSR
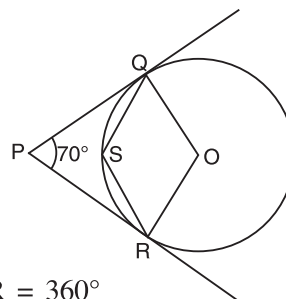
Ans. (a) Sum of the angles of the quadrilateral PQOR = 360°

$\Rightarrow$ $\angle$QPR + $\angle$PRO + $\angle$QOR + $\angle$OQP = 360°

$\Rightarrow$ $\qquad\qquad$ $70° + 90° + \angle QOR + 90° = 360°$

$\Rightarrow$ $\qquad\qquad\qquad\qquad$ **$\angle QOR = 110°$**

(b) $\qquad\qquad$ Reflex angle $\angle QOR = 2\angle QSR$

$\Rightarrow$ $\qquad\qquad\qquad$ $(360° - 110°) = 2\angle QSR$ $\quad$ *i.e.* $\quad$ **$\angle QSR = 125°$**

Question 3 :

(i) A bag contains 5 white, 2 red and 3 black balls. A ball is drawn at random. What is the probability that the ball drawn is a red ball ? $\qquad$ **[2]**

Ans. $\because$ $\qquad\qquad$ Probability of an event $= \dfrac{\text{Number of favourable events}}{\text{Total number of events}}$

$\Rightarrow$ $\qquad$ **Probability of drawing a red ball** $= \dfrac{2}{5+2+3} = \dfrac{2}{10} = \dfrac{1}{5}$

(ii) A solid cone of radius 5 cm and height 9 cm is melted and made into small cylinders of radius of 0·5 cm and height 1·5 cm. Find the number of cylinders so formed. $\qquad$ **[2]**

Ans. $\because$ $\quad$ The number of small cylinders formed $= \dfrac{\text{Volume of solid cone melted}}{\text{Volume of each cylinder formed}}$

$$= \dfrac{\dfrac{1}{3} \times \pi \times 5^2 \times 9}{\pi \times (0·5)^2 \times 1·5} = \mathbf{200}$$

(iii) Two lamp posts AB and CD each of height 100 m are on either side of the road. P is a point on the road between the two lamp posts. The angles of elevation of the top of the lamp posts from the point P are 60° and 40°. Find the distances PB and PD. $\qquad$ **[3]**
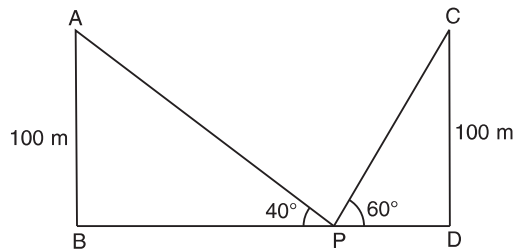
Ans. $\qquad$ In right-triangle PAB, $\tan 40° = \dfrac{100}{PB}$

$\Rightarrow$ $\qquad\qquad\qquad\qquad$ $0·8391 = \dfrac{100}{PB}$ $\quad$ *i.e.* $\quad$ **PB** $= \dfrac{100}{0·8391}$ m $=$ **119·18 m**

$\qquad$ In right-triangle PCD, $\quad \tan 60° = \dfrac{100}{PD}$ $\Rightarrow$ $1·7321 = \dfrac{100}{PD}$

i.e. $\qquad\qquad\qquad$ **PD** $= \dfrac{100}{1·7321}$ m $=$ **57·73 m**

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(iv) Marks obtained by 100 students in an examination are given below.　　　**[3]**

Marks	0-10	10-20	20-30	30-40	40-50	50-60
No. of students	5	15	20	28	20	12

Draw a histogram for the given data using a graph paper and find the mode.

Take 2 cm = 10 marks along one axis and 2 cm = 10 students along the other axis.

Ans.
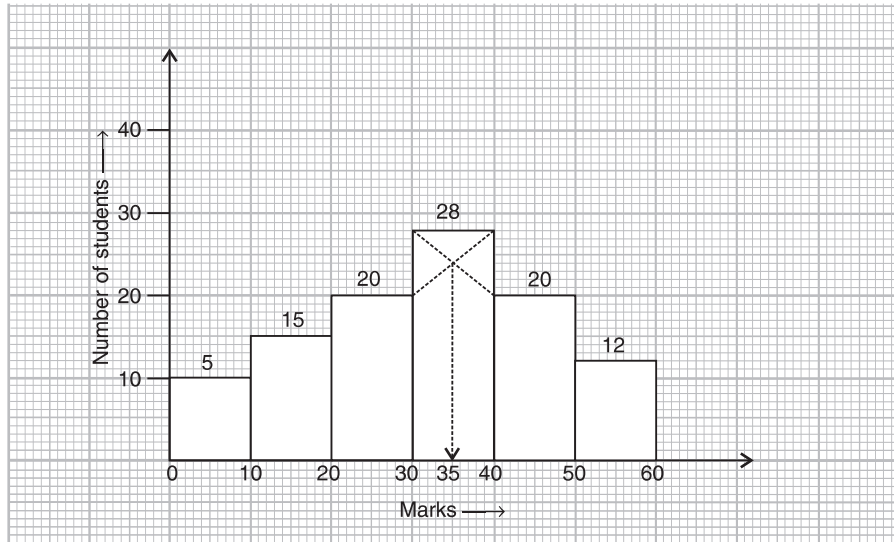
Mode = 35

Question **4 :**

(i) Find a point P which divides internally the line segment joining the points A(–3, 9) and B(1, –3) in the ratio 1 : 3.　　　**[2]**

Ans. $P_{(x)} = \dfrac{m_1 x_2 + m_2 x_1}{m_1 + m_2} = \dfrac{1 \times 1 + 3 \times -3}{1 + 3} = -2$

$P_{(y)} = \dfrac{m_1 y_2 + m_2 y_1}{m_1 + m_2} = \dfrac{1 \times -3 + 3 \times 9}{1 + 3} = 6$

$$A(-3, 9) \quad 1:3 = m_1 : m_2 \quad B(1, -3)$$
$$= (x_1, y_1) \qquad P(x, y) \qquad = (x_2, y_2)$$

∴ **P = (–2, 6)**

(ii) A letter of the word 'SECONDARY' is selected at random. What is the probability that the letter selected is not a vowel ?　　　**[2]**

Ans. The word SECONDARY has 3 vowels and 6 consonants

Number of favourable cases = 6 and

Total number of cases = 9

∴ **Required probability** $= \dfrac{6}{9} = \dfrac{2}{3}$

(iii) Use a graph paper for this question. Take 2 cm = 1 unit along both the axes. **[3]**

  (a) Plot the points
  A(0, 4), B(2, 2), C(5, 2), D(4, 0) and E(0, 0) is the origin.

  (b) Reflect B, C, D on the *y*-axis and name them as B′, C′ and D′ respectively.

  (c) Join the points ABCDD′C′B′ and A in order and give a geometrical name to the closed figure.
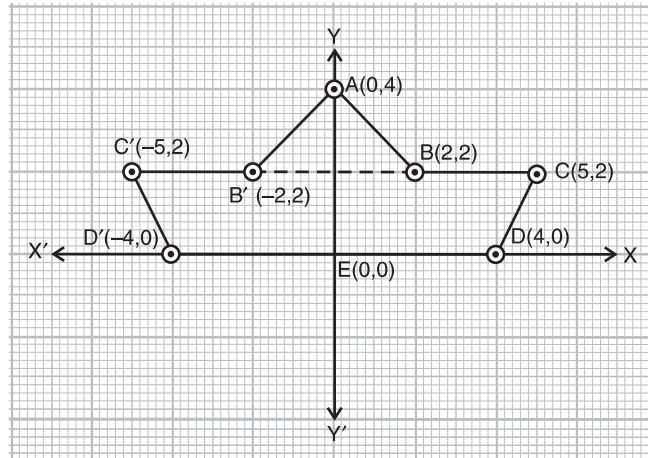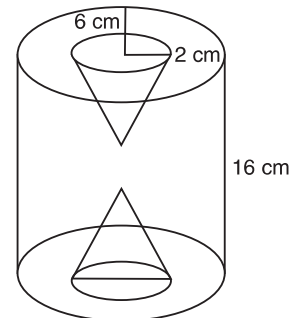
Ans.

Figure is like a ship

(iv) A solid wooden cylinder is of radius 6 cm and height 16 cm. Two cones each of radius 2 cm and height 6 cm are drilled out of the cylinder. Find the volume of the remaining solid. Take $\pi = \dfrac{22}{7}$ **[3]**

Ans. Volume of the remaining solid

= Volume of cylinder − 2 × volume of each cone

$= \pi \times 6^2 \times 16 - 2 \times \dfrac{1}{3}\pi \times 2^2 \times 6$

$= \dfrac{22}{7}[576 - 16]$ cm^3

$= \dfrac{22}{7} \times 560$ cm^3 = 22 × 80 cm^3 = **1760 cm³**

Question **5 :**
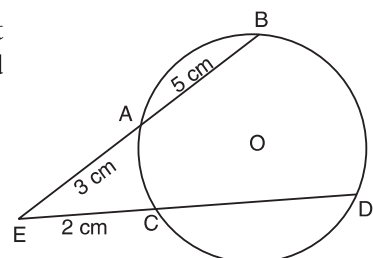
  (i) Two chords AB and CD of a circle intersect externally at E. If EC = 2 cm, EA = 3 cm and AB = 5 cm, find the length of CD. **[2]**

Ans. Let CD = *x* cm ⇒ ED = (2 + *x*) cm

  ∵   EC × ED = EA × EB

  ⇒   2 × (2 + *x*) = 3 × 8

  ⇒   2 + *x* = 12 and ***x* = 10**

(ii) Line AB is perpendicular to CD. Coordinates of B, C and D are respectively (4, 0), (0, −1) and (4, 3). **[2]**

Find :

(a) Slope of CD.

(b) Equation of AB.

A
D(4, 3)
B(4, 0)
C(0, −1)
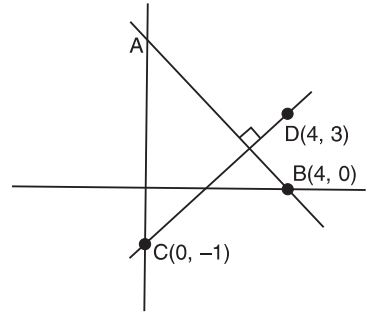
Ans. (a) Slope of CD = $\dfrac{3+1}{4-0}$ = **1**

(b) Slope of AB = $\dfrac{1}{-1}$ = −1

For AB : Slope m = −1 and (x_1, y_1) = (4, 0)

∴ **Equation of AB is**

⇒ $y - y_1 = m(x - x_1)$

⇒ $y - 0 = -1 \times (x - 4)$ ⇒ $x + y - 4 = 0$

(iii) Prove that : **[3]**

$$\frac{(1+\sin\theta)^2 + (1-\sin\theta)^2}{2\cos^2\theta} = \sec^2\theta + \tan^2\theta.$$

Ans. L.H.S. = $\dfrac{1+\sin^2\theta + 2\sin\theta + 1 + \sin^2\theta - 2\sin\theta}{2\cos^2\theta}$

= $\dfrac{2 + 2\sin^2\theta}{2\cos^2\theta}$

= $\dfrac{1}{\cos^2\theta}$ + $\dfrac{\sin^2\theta}{\cos^2\theta}$ = $\sec^2\theta + \tan^2\theta$

= **R.H.S.**   **Hence proved**

(iv) The mean of the following distribution is 50. Find the unknown frequency. **[3]**

Class Interval	0-20	20-40	40-60	60-80	80-100
Frequency	6	f	8	12	8

Ans.

C.I.	f	x	$f \times x$
0-20	6	10	60
20-40	f	30	$30f$
40-60	8	50	400
60-80	12	70	840
80-100	8	90	720
$\Sigma f = 34 + f$		$\Sigma f \times x = 2020 + 30f$	

Mean = 50

⇒ $\dfrac{\Sigma f \times x}{\Sigma f}$ = 50

⇒ $\dfrac{2020 + 30f}{34 + f}$ = 50

⇒ $2020 + 30f = 1700 + 50f$

⇒ $20f = 320$

∴ $f = \mathbf{16}$

Question **6 :**

(i) Prove that : [2]

$$1 + \frac{\tan^2\theta}{1+\sec\theta} = \sec\theta$$

Ans. L.H.S. $= 1 + \dfrac{\dfrac{\sin^2\theta}{\cos^2\theta}}{1+\dfrac{1}{\cos\theta}}$

$$= 1 + \frac{\dfrac{\sin^2\theta}{\cos^2\theta}}{\dfrac{\cos\theta+1}{\cos\theta}}$$

$$= 1 + \frac{\sin^2\theta}{\cos\theta\,(\cos\theta+1)}$$

$$= 1 + \frac{1-\cos^2\theta}{\cos\theta\,(\cos\theta+1)} = 1 + \frac{(1-\cos\theta)\,(1+\cos\theta)}{\cos\theta\,(\cos\theta+1)}$$

$$= 1 + \frac{1-\cos\theta}{\cos\theta}$$

$$= 1 + \sec\theta - 1 = \sec\theta$$

$$= \textbf{R.H.S.} \qquad \textbf{Hence proved}$$

(ii) In the given figure A, B, C and D are points on the circle with centre O. Given $\angle ABC = 62°$. [2]

Find :

(a) $\angle ADC$.

(b) $\angle CAB$.

Ans. (a) Since angles of the same segment are equal, $\angle\textbf{ADC} = \angle ABC = \textbf{62°}$

(b) Since angle of semi-circle is 90°, $\angle ACB = 90°$

In ΔABC :

$62° + 90° + \angle CAB = 180°$

$\Rightarrow \qquad\qquad \angle\textbf{CAB} = \textbf{28°}$

(iii) Find the equation of a line parallel to the line $2x + y - 7 = 0$ and passing through the intersection of the lines $x + y - 4 = 0$ and $2x - y = 8$. [3]

Ans. For the given line $2x + y - 7 = 0$

$$y = -2x + 7$$

$\Rightarrow$ its slope $= -2$ and the slope of its parallel is also -2.

On solving $x + y - 4 = 0$ and $2x - y = 8$, we get $x = 4$ and $y = 0$

For the required line :

$$m = -2 \text{ and } (x_1, y_1) = (4, 0)$$

$\therefore$ **Equation is :** $y - y_1 = m(x - x_1)$

$\Rightarrow \qquad\qquad y - 0 = -2(x - 4)$

$\Rightarrow \qquad\qquad \textbf{2x + y = 8}$

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(iv) Marks obtained by 40 students in an examination are given below. **[3]**

Marks	10-20	20-30	30-40	40-50	50-60	60-70
No. of students	3	8	14	9	4	2

Using graph paper draw an ogive and estimate the median marks. Take 2 cm = 10 marks along one axis and 2 cm = 5 students along the other axis.

Ans.

Marks C.I.	Number of students (Frequency)	Cumulative frequency
10-20	3	3
20-30	8	11
30-40	14	25
40-50	9	34
50-60	4	38
60-70	2	40

Median = 37